MOTION OF GAS BUBBLES IN A LIQUID UNDER THE INFLUENCE OF A TEMPERATURE GRADIENT

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ABSTRACT: This study deals with the motion of a gas bubble developing under the influence of surface-tension forces in an imponderable viscous liquid with a temperature gradient. A theory of steady-state motion of a bubble in a field with constant temperature gradient is given for the case of small Reynolds numbers. Experimental results that show qualitative agreement with the theory are presented.*

In an unbounded imponderable viscous liquid let there be a gas bubble of radius r_0. We shall consider the motion of a bubble so small that inertial forces can be neglected as compared with viscous forces. In a nonuniformly heated liquid, as a result of the temperature dependence of the surface tension, forces develop under whose influence the bubble may move.

We shall consider the steady motion of a bubble in a liquid medium with constant temperature gradient. It is known [1] that in this case in the bubble at rest the temperature gradient will also be constant. We shall assume that when the bubble moves the temperature gradient inside it preserves a constant value. We shall also neglect the dependence of viscosity on temperature.

At small Reynolds numbers the equations of motion of an incompressible viscous liquid have the form

$$\operatorname{grad} p = \eta \Delta v, \qquad \operatorname{div} v = 0. \tag{1}$$

We shall take a coordinate system such that the temperature gradient is directed along the z axis. Let the bubble move with constant velocity V along the z axis, which is obvious from considerations of symmetry. We pass to a coordinate system tied to the center of the bubble and consider the problem in spherical coordinates. In this case, in the usual notation, Eqs. (1), with allowance for symmetry about the z axis ($v_\varphi = 0$), have the form

$$\frac{\partial p}{\partial r} = \eta\left(\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 U_r}{\partial \theta^2} + \frac{2}{r}\frac{\partial U_r}{\partial r} + \frac{\operatorname{ctg}\theta}{r^2}\frac{\partial U_r}{\partial \theta} - \frac{2}{r^2}\frac{\partial U_\theta}{\partial \theta} - \frac{2U_r}{r^2} - \frac{2\operatorname{ctg}\theta}{r^2}U_\theta\right),$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = \eta\left(\frac{\partial^2 U_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 U_\theta}{\partial \theta^2} + \frac{2}{r}\frac{\partial U_\theta}{\partial r} + \frac{\operatorname{ctg}\theta}{r_2}\frac{\partial U_\theta}{\partial \theta} + \frac{2}{r^2}\frac{\partial U_r}{\partial \theta} - \frac{U_\theta}{r^2 \sin^2\theta}\right),$$

$$\frac{\partial U_r}{\partial r} + \frac{1}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{2U_r}{r} + \frac{U_\theta \operatorname{ctg}\theta}{r} = 0. \tag{2}$$

Assuming that the gas inside the bubble is an incompressible viscous fluid, we can write for its motion, equations analogous to (2). We shall, moreover, denote all the quantities relating to the gas by a prime. The solution of these equations must satisfy the following boundary conditions:

when $r \to \infty$

$$U_r = -V\cos\theta, \quad U_\theta = V\sin\theta, \quad p = p_0, \tag{3}$$

when $r = r_0$

$$U_r = U_r' = 0, \qquad U_\theta = U_\theta',$$

$$\eta\left(\frac{1}{r_0}\frac{\partial U_r}{\partial \theta} + \frac{\partial U_\theta}{\partial r} - \frac{U_\theta}{r_0}\right) = \eta'\left(\frac{1}{r_0}\frac{\partial U_r'}{\partial \theta} + \frac{\partial U_\theta'}{\partial r} - \frac{U_\theta'}{r_0}\right) - \frac{1}{r_0}\frac{\partial \sigma}{\partial \theta},$$

$$p - 2\eta\frac{\partial U_r}{\partial r} + \frac{2\sigma}{r_0} = p' - 2\eta'\frac{\partial U_r'}{\partial r}. \tag{4}$$

Here σ is the surface tension. In view of the above assumption about the constancy of dT/dz inside the bubble, the relation between σ and θ has the form

$$\sigma = \sigma_0 + \left(\frac{\partial \sigma}{\partial T}\right)\left(\frac{dT}{dz}\right) r_0 \cos\theta. \tag{5}$$

As in [2], we seek the solution of the problem in the form

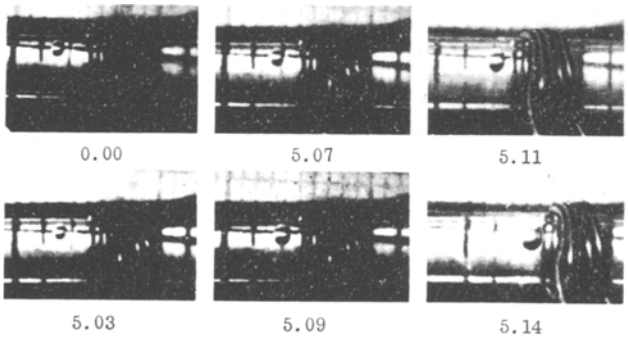
$$U_r = \left(A + \frac{B}{r} + \frac{C}{r^3}\right)\cos\theta,$$
$$U_\theta = -\left(A + \frac{1}{2}\frac{B}{r} - \frac{1C}{2r^3}\right)\sin\theta,$$
$$p = D + \eta\frac{B}{r^2}\cos\theta,$$
$$U_r' = (A' + B'r^2)\cos\theta,$$
$$U_\theta' = -(A' + 2B'r^2)\sin\theta,$$
$$p' = D' + \eta' 10 B' r\cos\theta. \tag{6}$$

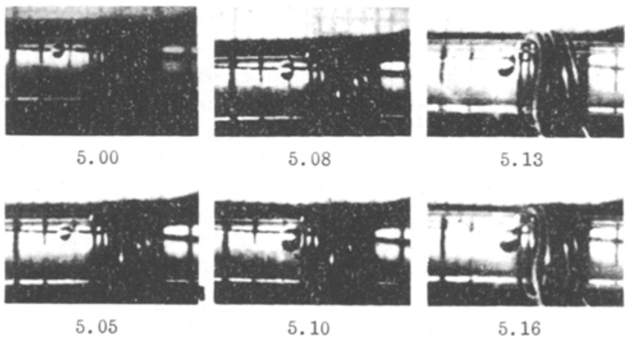
The constants and velocity V are found from the boundary conditions (3), (4), with allowance for (5).

As a result of computations, for the velocity of the bubble we get

$$V = -\frac{2}{3}\frac{r_0}{2\eta + 3\eta'}\left(\frac{\partial \sigma}{\partial T}\right)\frac{dT}{dz}. \tag{7}$$

*Editorial note. Before publication the authors notified the editors that they had recently become aware of another paper on the same theme: "The motion of bubbles in a vertical temperature gradient," by N. O. Young, L. S. Goldstein, and M. J. Block, J. of Fluid Mech., vol. 6, p. 3, 1959. However, for technical reasons it was not possible for the editors to accede to the authors' request to hold over the paper to permit comparison of the results.





Since $\partial\sigma/\partial T < 0$, it follows that the bubble velocity is directed in the sense of increase in temperature. Usually $\eta' \ll \eta$, and Eq. (7) can be written approximately in the form

$$V = -\frac{r_0}{3\eta}\left(\frac{\partial\sigma}{\partial T}\right)\frac{dT}{dz}. \qquad (8)$$

In the case of an air bubble in water we have

$$\eta = 0.01 \text{ g/cm} \cdot \text{sec}, \quad \partial\sigma/\partial T = -0.15 \text{ erg/cm}^2 \cdot \text{deg},$$

$$V = 15 r_0\, dT/dz, \quad [V] = \text{cm/sec} \quad [r_0] = \text{cm}, \quad [dT/dz] = \text{deg/cm}. \qquad (9)$$

The result obtained holds true if the Reynolds number

$$R = \frac{\rho r_0 V}{\eta} \ll 1. \qquad (10)$$

In the case in question this is equivalent to the condition

$$500 r_0^2 \frac{dT}{dz} \ll 1. \qquad (11)$$

We shall now present some experimental results. In view of the difficulties related with the presence of gravity, which causes convective motion of the liquid and rising of the bubbles, the experiment is only qualitative in character. Into a horizontal glass tube 2.5 mm in diameter and filled with distilled water, a bubble of air about 0.7 mm in diameter was introduced. Several millimeters away, outside the tube, we placed a nichrome spiral, by heating which we created a temperature gradient in the water. The motion of the bubble was registered by motion picture photography at a film speed of 300 frames/sec.

The results are shown in the figure. The bubble, initially at rest, begins to move 5-6 sec after the commencement of heating. As may be seen in the photographs, the air bubble, expanding as a result of evaporation, moves in the direction of increasing temperature. Thus, the qualitative result of the experiment coincides with the conclusion of the theory.

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